A Method for Forward Displacement Analysis of 3-RRP and 3-PRP Planar Parallel Manipulators

Sevasti Mitsi*, Konstantinos D. Bouzakis*, Gabriel Mansour*, Iulian Popescu**

*Mechanical Engineering Department, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece (Tel: 00302310 996043; e-mail: mitsi@eng.auth.gr).
**Faculty of Mechanics, University of Craiova, Romania

Abstract: The paper presents a method for the forward displacement analysis of 3-RP and 3-PR planar parallel manipulators including the RP-RP-RP third-class Assur group. The aim of this analysis is to find all possible configurations of the parallel planar mechanism for one given set of input joints values. The proposed method leads to a non-linear system of three equations with three unknown parameters. Using a successive elimination procedure, a polynomial equation of eighth order in one unknown is obtained. The real solutions of the polynomial equation correspond to the assembly modes of the planar parallel mechanism. The maximum number of the assembly modes of the investigated manipulators is two. Furthermore, a numerical application of the proposed method is presented.

Keywords: planar parallel manipulator, forward displacement analysis, Assur group.

1. INTRODUCTION

Parallel manipulators have some potential advantages over serial manipulators such as higher stiffness and better accuracy, higher velocities and accelerations, greater payload-to-weight ratio, possibility to locate actuators on the fixed base. However, they present limited workspace and multiple direct kinematic solutions (Tsai, 1999).

In the parallel planar manipulators all the moving links perform planar motions. Different methods for the direct kinematics of 3-DOF planar parallel manipulators have been developed by Gosselin et al. (1992), Mohammadi-Danial et al. (1993), Merlet (1996), and Kong and Gosselin (2001).

This paper presents a method for the forward displacement analysis of the 3-RRP and 3-PRP planar parallel manipulators, where R, P, R, P denote revolute, prismatic, actuated revolute and actuated prismatic joints, respectively. The developed method takes into account that the 3-DOF planar parallel manipulators under the study are multi-loop mechanisms with decoupled structure, where the input links are connected with the fixed base and with one third-class Assur group (Mitsi et al., 2003), (Mitsi et al., 2008).

The paper is organized as follow. Section II describes the procedure for the direct kinematics solution of planar parallel manipulators under the study. Section III presents a methodology for displacement analysis of the RP-RP-RP third-class Assur group. Finally, a numerical application and conclusions are given.

2. PROCEDURE FOR FORWARD DISPLACEMENT ANALYSIS OF THE MANIPULATORS

The 3-RRP and 3-PRP planar parallel manipulators (Fig. 1) are closed-loop mechanisms with three degrees of freedom, composed of three RRP and PRP planar kinematical chains respectively with identically topology, all connecting the fixed base O1O2O3 to the moving platform P1P2P3. The actuated joints are located on the fixed base and are revolute R and prismatic P, respectively. It is observed that the both planar parallel manipulators contain, except the fixed link and input links, a RP-RP-RP third-class Assur group.

The forward displacement analysis of the manipulators under the study may be formulated as follows: given the geometry of the links, the coordinates of the fixed base points O1, O2 and O3 and the set of input joints values (angles \( \phi_o \) and displacements \( s_o \)) find the pose of the moving platform P1P2P3.

The above mentioned planar parallel manipulators have a decoupled structure (Mitsi et al., 2003), where the external joints of the Assur group are connected with the known moving links (here input links). For the forward displacement analysis is used a modular method, where the constraint equations can be written and solved separately for each module, in hierarchical order defined by the mechanism structure. The forward displacement analysis procedure of the 3-RRP and 3-PRP planar parallel manipulators consists of the following steps:

Determination of the joints coordinates \( A_1, A_2, \) and \( A_3 \) with respect to the frame coordinate system of the mechanism:

\[
\begin{align*}
\mathbf{r}_i &= \begin{bmatrix} x_i & y_i \end{bmatrix} = \\
&= \begin{bmatrix} x_o + l_{0ai} \cos \phi_o & y_o + l_{0ai} \sin \phi_o \end{bmatrix} \quad (1)
\end{align*}
\]

for 3-RRP manipulator

\[
\begin{align*}
\mathbf{r}_i &= \begin{bmatrix} x_i & y_i \end{bmatrix} = \\
&= \begin{bmatrix} x_o + s_o \cos \phi_o & y_o + s_o \sin \phi_o \end{bmatrix} \quad (2)
\end{align*}
\]
A method for forward displacement analysis of 3-RRP and 3-PRP planar parallel manipulators

The Romanian Review Precision Mechanics, Optics & Mechatronics, 2011, No. 39

138

for 3-PRP manipulator, where i=1,2,3.

Determination of the RP-RP-RP third-class Assur group links pose with the aid of the methodology presented in next section.

The position analysis of this Assur group can be formulated as follows: for a given position of the external revolute joints $A_1(x_{A1},y_{A1}), A_2(x_{A2},y_{A2}), A_3(x_{A3},y_{A3})$, and a given set of input data $l_1, l_2, l_3$, and $\alpha_1, \alpha_2, \alpha_3$, find the position of the links or the position of auxiliary points $B_1(x_{B1},y_{B1}), B_2(x_{B2},y_{B2}),$ and $B_3(x_{B3},y_{B3})$.

Fig. 1. 3-RRP and 3-PRP planar parallel manipulators

3. POSITION ANALYSIS OF THE RP-RP-RP THIRD-CLASS ASSUR GROUP

In Fig. 2 a RP-RP-RP third-class Assur group with three internal and three external prismatic joints is illustrated. The symbolic form of the third-class Assur group is defined as: $A_1C_1-A_2C_2-A_3C_3$, where $A_iC_i$ (i=1,2,3) are the binary links, first and second symbol of the binary link correspond to external and internal joint respectively. The dimensions $P_1P_2$, $P_2P_3$, and $P_1P_3$ of the ternary link (moving platform) are given.

Considering triangles $A_1B_2A_2, A_2B_3A_3,$ and $A_3B_1A_1$ (Fig. 3), the following constraint equations can be written:

$$l_{A1A2}^2 = (x_{A2} - x_{A1})^2 + (y_{A2} - y_{A1})^2$$ (6)

$$l_{A2A3}^2 = (x_{A3} - x_{A2})^2 + (y_{A3} - y_{A2})^2$$ (7)

$$l_{A3A1}^2 = (x_{A1} - x_{A3})^2 + (y_{A1} - y_{A3})^2$$ (8)

After transformation, the constraint equations (3)-(5) take the form:

$$s_1^2 + s_2^2 + a_{11}s_1s_2 + a_{12}s_1 + a_{13}s_2 + a_{14} = 0$$ (9)

$$s_2^2 + s_3^2 + a_{21}s_2s_3 + a_{22}s_2 + a_{23}s_3 + a_{24} = 0$$ (10)

$$s_1^2 + s_3^2 + a_{31}s_1s_3 + a_{32}s_3 + a_{33}s_1 + a_{34} = 0$$ (11)

where the coefficients $a_{ij}$ (i=1,2,3 and j=1,2,3,4) depend on the geometry data and the position of external joints $A_1, A_2,$ and $A_3$. Detailed expressions of these coefficients are listed in the Appendix A.
The system of nonlinear equations (9)-(11) is solved through successive application of the resultant method (Van der Waerden, 1991). Since the variable \( s_2 \) is present only in (9) and (10), using the Sylvester theorem, one can eliminate \( s_2 \) from these equations yielding:

\[
F_{12}(s_1, s_3) = 0
\]  

(12)

Again, using Sylvester theorem and eliminating the variable \( s_3 \) from (11) and (12) yields the eighth order polynomial equation in the only unknown \( s_1 \):

\[
\sum_{i=0}^{8} P_i s_1^i = 0
\]  

(13)

The coefficients \( P_i \) (\( i=0,1,...,8 \)) depend only on the Assur group data. Equation (13) provides eight roots for \( s_1 \) in the complex field from which maximum two real solutions lead to real solutions for displacements \( s_2 \) and \( s_3 \) (Pelecudi, 1975), (Chung, 2005). Therefore, the maximum number of the assembly modes of the RP-RP-RP Assur group with three external revolute joints and three internal prismatic joints is two. Furthermore, the maximum number of the assembly modes of the 3-RRP and 3-PRP planar parallel manipulators is two.

For every real solution for displacement \( s_1 \), the coordinates of the points \( B_1, B_2, \) and \( B_3 \) are determined.

The coordinates of the point \( B_1 \) are (see Fig. 3):

\[
r_{B_1} = \begin{bmatrix} x_{B_1} \\ y_{B_1} \end{bmatrix} = \begin{bmatrix} x_{A_1} + s_1 \cos \theta_1 + s_3 \sin \phi_1 \\ y_{A_1} + s_3 \sin \theta_1 \end{bmatrix}
\]  

(14)

where: \( \phi_1 = \theta_1 + \gamma_1 \)

with \( \theta_1 = \arctan\left(\frac{y_{A_2}-y_{A_1}}{x_{A_2}-x_{A_1}}\right) \) and angle \( \gamma_1 \) evaluated by considering the law of sines for triangle \( A_1B_2A_2 \):

\[
sin \gamma_1 = s_2 \sin \alpha_2 / l_{A_1A_2}
\]  

(16)

Furthermore, the coordinates of the points \( B_2 \) and \( B_3 \) are respectively:

\[
r_{B_2} = \begin{bmatrix} x_{B_2} \\ y_{B_2} \end{bmatrix} = \begin{bmatrix} x_{B_1} + l_1 \cos \phi_1 + l_2 \sin \phi_1 \\ y_{B_1} + l_1 \sin \phi_1 + l_2 \sin \phi_1 \end{bmatrix}
\]  

(17)

\[
r_{B_3} = \begin{bmatrix} x_{B_3} \\ y_{B_3} \end{bmatrix} = \begin{bmatrix} x_{B_1} + l_3 \cos(\phi_1 + \alpha_1) + l_2 \sin(\phi_1 + \alpha_1) \\ y_{B_1} + l_3 \sin(\phi_1 + \alpha_1) + l_2 \sin(\phi_1 + \alpha_1) \end{bmatrix}
\]  

(18)

4. NUMERICAL APPLICATION

In this section the proposed procedure is applied to a symmetrical planar 3-RRP parallel manipulators, where \( O_1O_2O_2=O_2O_3=O_3O_1, O_1A_1=O_2A_2=O_3A_3, l_1=l_2=l_3, \) and \( \phi_1=\alpha_2=\alpha_3=0 \) (see Fig. 1). The geometrical data and the input links position angle \( \phi_0 \) (\( i=1,2,3 \)) are given in the Table 1. For the specific geometry here considered, by solving the polynomial equation (13), two real solutions of the displacement \( s_1 \) are obtained. For each real solutions of the \( s_1 \) (see Table 1), the displacements \( s_2, s_3 \), and the coordinates of the points \( B_1, B_2, \) and \( B_3 \) are calculated. The pose and orientation of the moving platform is defined by using the coordinates of the points \( B_1 \) and \( B_2 \) given in the Table 2. The corresponding two assembly modes of the planar 3-RRP parallel manipulator are illustrated in Fig. 4.

Table 1. Data and solutions of the 3-RRP parallel manipulator

<table>
<thead>
<tr>
<th>Config.</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-77.5558</td>
<td>-103.5205</td>
<td>-79.5285</td>
</tr>
<tr>
<td>2</td>
<td>42.7405</td>
<td>62.9381</td>
<td>34.9262</td>
</tr>
</tbody>
</table>

Table 2. Coordinates of the moving platform points

<table>
<thead>
<tr>
<th>Config.</th>
<th>Coordinates of moving platform points ( B_1, B_2, ) and ( B_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x_{B_1} ) \hspace{1cm} ( y_{B_1} ) \hspace{1cm} ( x_{B_2} ) \hspace{1cm} ( y_{B_2} ) \hspace{1cm} ( x_{B_3} ) \hspace{1cm} ( y_{B_3} )</td>
</tr>
<tr>
<td>1</td>
<td>63.6963 \hspace{1cm} 36.0020 \hspace{1cm} 74.8446 \hspace{1cm} 76.3792 \hspace{1cm} 74.8446 \hspace{1cm} 34.8662</td>
</tr>
<tr>
<td>2</td>
<td>48.2205 \hspace{1cm} 83.9904 \hspace{1cm} 50.6010 \hspace{1cm} 76.3792</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

Compared with other methods, the proposed procedure solves simultaneously the forward kinematics analysis of two planar parallel manipulators 3-RRP and 3-PRP. Furthermore, the procedure can be extended for direct kinematics analysis of 3-DOF planar parallel manipulators with actuated joints located on the fixed base as 3-RRR, 3-PRR including a RR-
A method for forward displacement analysis of 3-RRP and 3-PRP planar parallel manipulators

RR-RR Assur group and 3-RPR, 3-PPR including a PR-PR-PR Assur group.

Fig. 4. Assembly modes of the 3-RRP parallel manipulator

REFERENCES


Appendix A.

The coefficients $a_{ij}$ (i=1,2,3 and j=1,2,3,4) of (9) - (11) are:

\[
\begin{align*}
a_{11} &= 2 \cos \alpha_2 \\
a_{12} &= 2l_1 \\
a_{13} &= 2l_1 \cos \alpha_2 \\
a_{14} &= l_1^2 - l_{81B2}^2 \\
a_{21} &= 2 \cos \alpha_3 \\
a_{22} &= 2l_2 \\
a_{23} &= 2l_2 \cos \alpha_3 \\
a_{24} &= l_2^2 - l_{82B3}^2 \\
a_{31} &= 2 \cos \alpha_4 \\
a_{32} &= 2l_3 \\
a_{33} &= 2l_3 \cos \alpha_4 \\
a_{34} &= l_3^2 - l_{83B1}^2
\end{align*}
\]