MODELING AND EXPERIMENTAL VALIDATION OF THE EFFECTS OF EVAPORATION OF WATER DROPLETS ON MICRO-CANTILEVER BEAMS SUBJECTED TO LARGE DEFLECTION

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Abstract – The main objective of this investigation is to evaluate the performance of microcantilever beams as sensors for detecting the bio-chemical reaction between an analyte and a reaction agent. In this study a droplet placed on the microcantilever beam (MCLB) is mathematically modelled. Sitting area of droplet base, the volume, the peripheral length, the deviation of mass center and force and moment distribution of the droplet were analytically obtained. The effect of the superficial tension, weight and Laplace pressure on the MCLB due to the evaporation of the droplet were separately investigated. Deflections of the MCLB illustrated is assumed with in large deflection theory and the solution is found numerically using the finite element method. Total moments and total displacements of all effects were evaluated in the calculations. Results obtained from the total moments showed good agreement with the experiment.

Keywords – Water evaporation, droplets, micro-cantilever beams

1. Introduction:

Droplet evaporation positioned on the MCLB or on surfaces is of high importance in biomedical and industrial application. Spray painting, cold spray coatings, food and drug industries are among the main beneficiary of the identified phenomena. The sensors base on microcantilever offer not only improved dynamic response, greatly reduced size, high precision, and increased reliability but also are easy to fabricate and exhibit extremely small thermal mass and low cost compared with the conventional sensors [1]. Some enzymatic reactions can be detected. Moreover, highly sensitive biochemical sensor applications such as DNA, antibody, and restriction endonuclease assays can be investigated based on microcantilever essays. The study realised with the microcantilever beam dimensions of 100–350 mm length, 20–40 mm width, and 0.6–1 mm thickness presented just one possible detection mode of microcantilevers and by reducing the thickness of the cantilever, the sensitivity can be improved by a few orders of magnitude [1].

The evaporation dynamics of water droplets from the surfaces of cantilevers were also investigated. After placing a water droplets on cantilevers, variations in the resonance frequency and deflection during evaporation were related to the changes in mass and stress of the cantilever, respectively [6].

A mathematical model of droplet evaporation validated by experiments was proposed by Dunn in 2007 [7]. In this study we investigated the effects of evaporation of water droplet sitting on the MCLB on the deflection of the deformation using the large deflection theory.

Experimental results indicate that evaporation of the last thin layer of water is significantly slower than the meniscus of the droplet, which can be due to surface forces [2]. The FEM model was developed for cantilever bending and thus it was confirmed by experimental results [3]. The contact angle and radius of the evaporating droplets was monitored with video microscopy by an AFM-like setup [4].

The study about the role of vertical component of surface tension of a water droplet on the deformation of membranes and microcantilevers found with the aid of numerical simulations using the Mooney–Rivlin (MR) model and the linear elastic constitutive relation, respectively [5].

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The droplet will interact with the cantilever through the gravitational effect and through the superficial tension. Moreover, the fluid in the droplet will evaporate in time and thus, the contact these areas as well as the weight of the droplet will reduce. The work below presents the results of investigations on the phenomena. The proposed model yields good results as it accurately matches the experimental results.

2. The Mathematical Model:

The model assumes the droplet shape as a sphere cap sitting on the MCLB as shown in Figure 1. Here \( L, b, t \) are length, width and thickness of MCLB respectively. The contact surface, teh contour, the volume and the c.g. position are the important parameters in establishing the gravitational effect on the MCLB.

In Figure 2, front (b), right (a) and top views (c, d) of droplet sitting on the MCLB are shown. Taking into account for \( XOY, X'O'Y' \) frames in Figure 2, one can describe \( R, a \) and \( S \) as the diameter of entire drop, let its radius and peripheral circumference sitting on the MCLB, respectively. \( \zeta, \varphi \) and \( \varphi \) are special lengths and angle due to the geometry of the model (Equations 1-3) and \( x \) is measured from \( l_o \).

\[
\zeta = \sqrt{a^2 - (x - a)^2} \tag{1}
\]
\[
q = \sqrt{R^2 - a^2} \tag{2}
\]
\[
\varphi = \tan^{-1} \left( \frac{x - a}{\zeta} \right) \tag{3}
\]

One can write the equations of droplet sitting on the MCLB as described by Equation (4) in \( X'O'Y' \) plane.

\[
Z, Z' \tag{a}
\]
\[
Y' \tag{b}
\]
\[
X' \tag{c}
\]
\[
X, Y, Z \tag{d}
\]
\[ X^2 + Y^2 = a^2 \]  

(4)

Integrating (4) peripheral (S), contact area (A) and volume (V) of the droplet can be written in equations (5-10).

Peripheral of droplet:

\[ S = \int_{-a}^{x-a} \sqrt{1 + \left( \frac{dY'}{dX'} \right)^2} dX' \]

(5)

\[ S = 2a[\varphi + \frac{\pi}{2}] \]

(6)

The volume of the droplet is expressed by:

\[ V = \int_{-a}^{x-a} [(R^2-x^2)\tan^{-1}\left(\frac{\sqrt{a^2-x^2}}{q}\right) - q\sqrt{a^2-x^2}] dX \]

(7)

\[ V = \frac{1}{3} \left( q^2 - 3\eta R^2 \right) \varphi - \left( x - a \right) \left( 2\eta \zeta + (-3) R^2 \right) + \left( x - a \right)^2 \tan^{-1}\left( \frac{\zeta}{q} \right) - tRtan^{-1}^{-1} \left( \frac{q(x-a)}{iR\zeta} \right) - \frac{\pi}{6} (-q^2 + 3qR^2 - 2R^3) \]

(8)

Using equation 4, the mass center of droplet can be obtained in Eq11. Given of the symmetry principle, the center of mass must lie on the x axis and thus there is no need to calculate its position, so \( \bar{y} = 0 \). After simplifications, mass center position can be written as in Eq. 13.

\[ \bar{x} = \frac{A}{1} \int_{-a}^{x-a} \frac{1}{2} X^2 dY' \]

(9)

\[ \bar{x} = \frac{1}{A} \int_{-a}^{x-a} \frac{1}{2} (\alpha^2 - Y'^2 - (a - x)^2) dY' \]

(10)

\[ \bar{x} = \frac{1}{A} \left( \frac{1}{3} \zeta^2 + 2a \alpha \zeta - x^2 \zeta \right) \]

(11)

Considering Table 1, the peripheral length (S), the sitting area (A), the volume (V) and the mass center of droplet sitting on the MCLB are illustrated in Figure 3 (a,b,c,d).

Figure 3: Some geometric properties of the droplet along the length of MCLB: the peripheral length (a), the sitting area (b), the volume (c) and the mass center position (d).
3. The Bending Moments on the MCLB:

One can invstigate moment distributions in the three regions on the MCLB when evaporation of the droplet occurs. In Figure 3, the three regions and $M_1$, $M_2$ and $M_3$ moments are illustrated under the equivalent force $F$ effect by droplet. Here $T$ is shear force. The measurement from the tip point yields a major advantage in illustrating the MCLB deflection.

![Figure 4: Moment distribution on the MCLB.](image)

These moments can be written as in Eq.14.

$$M = \begin{cases} M_3 & \text{if}, \quad 0 \leq x \leq l_0 \\ M_2 & \text{if}, \quad l_0 \leq x \leq l_0 + 2\alpha \\ M_1 & \text{if}, \quad l_0 + 2\alpha \leq x \leq L \end{cases}$$  \hspace{1cm} (14)

We shall consider the three effects which are superficial ($\gamma$), Laplace pressure ($p$) and gravity ($w$) on the MCLB. Some of the moments influence the deflection of MCLB. These moments are due to: the superficial tension, Laplace pressure and weight [2]. Moments and shear forces of the above causes were obtained separately. In Figures (5-8), left column shows moment and shear force under the droplet and right column shows moment and shear force for the entire beam.

3.1. The Superficial Tension

There are two components of superficial tension vertical ($\gamma_y$) and horizontal ($\gamma_x$).

3.2. The Vertical Component

The vertical component, shear force and moment of droplet can be seen in Eq. (15), (16) and (17).

$$\gamma_y = \gamma \sin(\theta)$$  \hspace{1cm} (15)

$$F_\gamma = S \gamma_y$$  \hspace{1cm} (16)

$$M_\gamma = (x - \bar{x})F_\gamma$$  \hspace{1cm} (17)

In Figure 5, considering parameters in Table 1, shear force (a,b) and moment patterns (c,d) are illustrated. The droplet is placed between 0.01 and 0.003 on the MCLB. Closing to the fix point of MLCB there is no effects after the droplet and thus shear force keeps the same level (Fig.5c).
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Figure 5: Superficial force under the effect of the droplet (a), moment due to the superficial force under the droplet (b), shear force (c) and moment (d) for the entire beam.

Table 1. Design parameters of droplet and MCLB.

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>b (mm)</th>
<th>t (μm)</th>
<th>R (mm)</th>
<th>L (mm)</th>
<th>l₀ (mm)</th>
<th>γ (N/m)</th>
<th>E_PVDF (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>20</td>
<td>3.5</td>
<td>4.62</td>
<td>1</td>
<td>0.0072</td>
<td>2.27</td>
</tr>
</tbody>
</table>

3.3. The Horizontal Component

Horizontal components of superficial tension which are parallel to the original surface of the MCLB and the corresponding shear force and moments are given Eq. (18), (19) and (20).

\[ F_{yx} = S_\gamma_x \sin(\varphi) \]  

\[ M_{yx} = (x - \bar{x})F_{yx} \]  

Figure 6: Horizontal component of the superficial force (a) and moment (b) under the droplet, force (c) and moment (d) for the entire beam.
3.4. The Laplace Pressure

The Laplace pressure represents the pressure difference between the inside and the outside of the curved surface of the droplet. The pressure difference is caused by the surface tension of the interface between liquid and gas. The pressure difference, the force and the moment on the MCLB are given below.

\[ p = \frac{2 \gamma \sigma}{a} \]  

(21)

\[ F_p = -pA \]  

(22)

\[ M_p = (x - \bar{x})F_p \]  

(23)

In Figure 7, shear force (a,b) and moment patterns (c,d) are illustrated. Because there is no other effect except for the Laplace one in the interval strats from the point where the droplet end to the fix point of MCLB, shear force goes on a straight line (Fig.7c).

3.5. The Weight Effect

\[ F_w \] and \[ M_w \] are shear force and moments as effect of the weight of the droplet. Here, \( \rho \) is the density of the fluid and \( g \) is gravitational acceleration.

\[ F_w = -\rho g V \]  

(24)

\[ M_w = (x - \bar{x})F_w \]  

(25)

In Figure 8, shear force (a,b) and moment patterns (c,d) are illustrated. Because there is no other effect except for the gravity one in the interval strats from the point where the droplet end to the fix point of MCLB, shear force goes on a straight line (Fig.8c).

4. The Numerical Solution

\( M \) is total moment in Eq.26

\[ M = M_x + M_{y_2} + M_p + M_w \]  

(26)

Let’s call \( u \) as integration of \( M \) along the length of the beam:

\[ u = \int \frac{M}{EI} \, dx + C_1 \]  

(27)

The rotation and the deflection of the MCLB is as given in (28) and (29) respectively \[8\]. Here \( C_1 \) nd \( C_2 \) integral constants that depend on the boundry conditions.

\[ \theta = \frac{dy}{dx} = \frac{u}{\sqrt{-(u + C_1)^2 + (EI)^2}} \]  

(28)

\[ y = \int \theta \, dx + C_2 \]  

(29)

Regions 1 and 2 are in Fig.4 divided into 3 parts. Hence \( n = 3 \). Now there are 6 subregions of interet. \( \Lambda \) and \( \Gamma \) (Eq 30, 31) are increments for each region. \( u_{11} \) and \( u_{21} \) (Eq 32, 33) are moment integrals of the first subregion of first region and the first subregion of second region respectively.

\[ \Lambda = \frac{1}{n} (L - l_2 - 2a) \]  

(30)

\[ \Gamma = \frac{2a}{n} \]  

(31)
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After numerical calculation of all six angles, one can obtain the deflections in Eq (29) under the four effects.

5. The Experimental Study

Deflection of the micro cantilever beam (MCLB) was measured employing Wyko NT1100 optical profiling system. This system is a type of microscope which employs non-contact surface metrology technique. The experimental set-up is shown in Figure 9. The micro-cantilevers used in this work were fabricated as sandwich of layers of Molybdenum (Mo), polyvinylidenefluoride (PVDF), and Mo. The thickness of Mo layers (upper-lower) and PVDF layer (in the middle of the MCLB) are 1 \( \mu \text{m} \) and 18 \( \mu \text{m} \), respectively. The length L, width b and total thickness t of the micro-cantilever are 4.62 mm, 2 mm and 20 \( \mu \text{m} \), respectively. The mechanical property of PVDF are as follows: density 1,780 kg/m\(^3\), Young’s modulus 2.27 GPa and Possion ratios 0.225. The mechanical property of Mo are as following: density 10,300 kg/m\(^3\), Young’s modulus 275.9 GPa, poission ratio 0.32. The droplet volume was 1\( \mu \text{l} \). MCLB was clamped strongly between two metal slides, as shown in Figure 9.

Figure 9: The view of MCLB and droplet.

1. The Results

Figure 10 shows four effects on MCLB surface: vertical and horizontal component of superficial tension, Laplace and weight. While the vertical component and Laplace pressure effects present the largest deflections but the smallest one is caused by horizontal component.

Figure 10: Four effects produced by a droplet on the MCLB. Vertical (a) and horizontal (b) effects of superficial tension, Laplace pressure (c) and weight effects (d).
Total Deflection: There are two ways to figure out the total deflection: first is to sum up all moments and then assume it as only one moment $\Sigma M$, and the second is to sum up all deflections $\Sigma y$. As seen in Figure 11, keeping all parameters, larger deflection occurs in second case. We considered only first method in this paper. Both curves obtained in quadratic form (as polynomial) to easy show the difference.

![Figure 11: Red line is the deflection due to the sum of the moments, blue line is the deflection due to the sum of the displacements.](image1)

As the rotation at any point of the cantilever is closely related to the deflection of the cantilever, the deflection for four constant rotations selected based on the observation of the phenomena associated with the motion of the beam when loaded with droplets is carried out. Figure 12 shows the deflection of the cantilever for the four selected situations.

![Figure 12: Deflection of cantilever beam for values of $\gamma$.](image2)

In Figure 13, two experimental case are illustrated. In first one profile can be seen when droplet sit to the MCLB face (blue line), in second case maximum position (red line) of MCLB can bee seen during the evaporation process.

![Figure 13: Two case for MCLB profile: when droplet sticks to surface (blue line) and after evaporation has occurred](image3)

Figure 14 illustrates the comparison of experimental and numerical results. For $N/m$, we find the best match with the values obtained from the experiment.

![Figure 14: Comparison of MCLB deflections: numerical (blue line) and experimental one (red line)](image4)

6. Conclusion and Discussion

Large deflection theory is affected by non-linearity. Because of this, different effects cannot be superimposed. On the contrary, we investigated the MCLB deflection in two ways: using total moments (1) and total displacements (2).

- Under the assumption of total displacements it is found that the resultant deflection is smaller (Fig.11).
- However, under the assumption of total displacements the MCLB profile is not closer to the real profile. The cantilever goes down as if there is no superficial tension effect.
- As seen in Fig. 14 there is a spike close to the fix point of the MCLB and the experimental and numerical results are not close to each other. However, one can see...
the difference when the droplet is positioned to the MCLB surface (Fig.13). This may be due to the effect the hardening at both side of the MCLB as well as the imperfect surface.

- The model provides a good match at tip point.
- To achieve better match, one may consider to use very high superficial tension 6.081 N/m. This is possible only when use large deflection theory.

7. References

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