NUMERICAL DETERMINATION OF THE STATE OF TENSION IN NORMAL ELASTIC CONTACT BETWEEN BODIES LINED TOROIDAL SURFACES

Prof. Ing. Drd. Daniela Gheorghescu, Colegiul Național Anastasescu,
Str. Republicii, Nr.9-11, Roșiorii de Vede, Județ Teleorman

Abstract – In this report underline the general theoretical and experimental numerical determination of the state of stress at the contact of bodies lined the toroidal surface, analyzing the correlation between numerical model and conjugate gradient method to solve the analytical model radio. By running MATLAB computer programs, based on mathematical relations, the strain diagrams are plotted for different contact pairs of materials, forces of pressure, size and roughness of contact surfaces, causing the state of tension on the elliptical area contact under elliptical contact area and the central axis of contact.

Key words – analytical calculation, blood pressure, blood equivalent, elastic contact, elliptical area, numerical calculation

1. Introduction

Tor is the area generated by rotating a circle around an axis contained in the plane circle and outside it. Are determined by analytical representations of curves and surfaces in contact named principal in the first point of contact. Numerical modeling of the problem is the following: balance equation is outside the network is uniform 47x23 mesh nodes, is used optimally sizing estimated contact area, the influence coefficients are calculated analytically and the system pressure is solved conjugate gradient method. Elastic contact elements (analytic, numeric and relative error) are presented in Table 1.

Table 1 Elastic contact elements

<table>
<thead>
<tr>
<th>Type</th>
<th>Pressure results [GPa]</th>
<th>Near normal [m]</th>
<th>Semiaxa large ellipse [m]</th>
<th>Semiaxa small ellipse [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>analytical</td>
<td>3.91312</td>
<td>5.25891</td>
<td>2.19377</td>
<td>5.56059</td>
</tr>
<tr>
<td>numerical</td>
<td>3.91406</td>
<td>e-06</td>
<td>e-04</td>
<td>e-05</td>
</tr>
<tr>
<td></td>
<td>5.25775</td>
<td>e-06</td>
<td>2.19377</td>
<td>5.56059</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>e-04</td>
<td>e-05</td>
</tr>
<tr>
<td>Relative error</td>
<td>0.02414 %</td>
<td>0.02209 %</td>
<td>2.44 %</td>
<td>2.44 %</td>
</tr>
</tbody>
</table>

Analyzing the data in the table, there is a very good match between the two models. If the maximum contact pressure and normal approach, the relative error is recorded with values less than 0.25 %. Next, the graphics are the results of validation.
Numerical determination of the state of tension in normal elastic contact between bodies lined toroidal surfaces

To calculate the state of elastic tension points semispațiului Hertz potential method was used. General functions are defined by simple layer potential, ie logarithmic, [12], [1], [2],[3]. Using Papkovici-Boussinesq representation, displacement vector components are expressed through derivatives of potential functions. Stress tensor components are based on the generalized Hooke's law. Potential functions and their derivatives, determined analytically by Love, [12], are presented by other authors, Hills Nouell, Sackfield, cited by Cretu, [1].

Corresponding formulas are adapted specifically related to the algorithm implemented in MATLAB programming environment, taking into account the type of meshing and contact of the estimated range of results obtained in determining the normal elastic contact elements.

Numerical determination of the state of tension - variant II

This possibility of determining the state of tension produced in the bodies in contact, involves the following steps:
- to write expressions tensions produced by concentrated load applied normal to the plane of the adjacent elastic semispațiului (Boussinesq's problem);
- determine the elastic stresses produced semispațiul a load distributed over an area adjacent D plan (by the principle of overlapping effects);
- additivity property applies to the integral double the areas of integration
- Integrals that appear in the previous expressions are approximated by the corresponding integral sums. [8],[9],[11]

Validation status tensions elliptical radio contact

Based on the general algorithm, automated procedures were performed three:
- tensa1 - Analytical calculation of stress tensor components
- tensa1 - Numerical determination of the state of tension, variant I (analytical calculation of integrals involved in expressions of potential functions);
- tensa2 - Numerical determination of the state of tension, variant II (approximating the integrals involved in expressions of potential functions by the corresponding integral sums).

Were determined by numerical tensions of bodies in contact the following: the elliptic area under the contact area at a certain distance from the adjacent plane semispațiului elastic, and the central axis of contact.

To validate the state of tension in elliptical radio contact is considered an elastic contact between two bodies lined toroidal surfaces, normally charged with the task $Q = 100 \text{ N}$.

State of stress on elliptical contact area

By running computer programs tensa1.m ($Z_{ OBS } = 0 \text{ program for determining the tension in the elliptical contact area}$) and tensa1.m, to obtain comparative results presented in Table 2. The proposed numerical model and computer code associated tensor components lead to voltage values are in good agreement with the theory offered by radio. [4], [6],[14],[15]
Numerical determination of the state of tension in normal elastic contact between bodies lined toroidal surfaces

Table 2, stresses the elliptical contact area - dimensionless

<table>
<thead>
<tr>
<th>Maximum</th>
<th>σ_1/p₀</th>
<th>σ_2/p₀</th>
<th>σ_3/p₀</th>
<th>τ_{xy}/p₀</th>
<th>HMH/p₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>analytical</td>
<td>0.68089</td>
<td>0.91911</td>
<td>1</td>
<td>0.07147</td>
<td>0.28730</td>
</tr>
<tr>
<td>number (version I)</td>
<td>0.68092</td>
<td>0.91908</td>
<td>1</td>
<td>0.07121</td>
<td>0.28730</td>
</tr>
<tr>
<td>Relative error</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td></td>
<td>0.0046</td>
<td>0.0034</td>
<td>0   %</td>
<td>0.3638</td>
<td>0.0137</td>
</tr>
</tbody>
</table>

Consistency determinations numerical and analytical stress tensor components elliptical contact area is suggested, for example, in some graphics, which outlined the size of the tensions (unsigned).

Fig. 4 Tangential tension τ_{xy}/p₀ - analytically, that number

Fig. 5 Equivalent voltage HMH/p₀
a) the large axis of contact ellipse
b) the small axis of contact ellipse

Fig. 6 Normal tension axis larger than that of the axis of contact ellipse, analytical and numerical

State of tension in elliptical contact area

Determine the tension and pressure tensor components Huber – Missis - Hencky equivalent to points located in a plane parallel to the adjacent elastic semispace, at distance z_{obs} = 0.8b = 4.45 · 10^{-5} m, where b is small semiaxix contact ellipse.
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Some graphical modeling matches are listed below.[13]

Fig. 7 Normal blood $\sigma_x/P_0$ plan $z = 0.8 \times b$
   a) the large axis of contact ellipse
   b) the small axis of contact ellipse

Fig. 8 Normal blood $\sigma_y/P_0$ plan $z = 0.8 \times b$
   a) the large axis of contact ellipse
   b) the small axis of contact ellipse

Fig. 9 Normal blood $\sigma_z/P_0$ in plane $z = 0.8 \times b$
   a) the large axis of contact ellipse
   b) the small axis of contact ellipse

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Fig. 10 Tangential tension $\tau_{xy}/p_0$ - spatial distribution in plane $z = 0.8 * b$

a) Analytical    b) number, variant I

Fig. 11 Tangential tension $\tau_{yz}/p_0$ - spatial distribution in plane $z = 0.8 * b$

a) Analytical    b) number, variant I

Fig. 12 Equivalent voltage $HMH/p_0$ - in plane $z = 0.8 * b$

a) the large axis of contact ellipse
b) the small axis of contact ellipse
By running computer programs and tens1.m tensn2.m associated stress tensor determination and blood components Huber-Mises-Hencky equivalent, numerically by variant II (approximating the integrals by the full amount) and analytically, to obtain the results consistent.

**State of stress on the central axis of contact**

The points located on the z axis (center line of contact), normal stresses are principal normal stress, tangential stress as $\tau_{xx}$, $\tau_{yy}$, $\tau_{zz}$, cancels its points. Tensa1.m tensn1.m computer codes and help them to determine the tension on the central axis of the contact, using a range of values with constant step interval of z axis, where b is small semiaxa contact ellipse. In Fig.14 is shown graphically in depth development of the main normal stress and tension Huber-Mises-Hencky equivalent, determined analytically and numerically.[7],[9].

Fig.14 Variation of normal stress axis z a principaland equivalent voltage HMH, number $[o, *, +, x]$ and analytical $[---]$

Tangential stresses are plotted principal normal stresses and strains corresponding to octahedral, numerically and analytically.

Fig.15 Z a axis voltage variation main tangential $\tau_1$, respectively $\tau_2$, number $[o, *, +]$ and analytical $[---]$

Fig.16 Z a axis voltage variation main tangential $\tau_3$, number $[o, *, +]$ and analytical $[---]$

Fig.17 Z a axis voltage variation normality $\sigma_{11}, \sigma_{22}, \sigma_{33}$ respectively octahedral stress, number $[o, *, +]$ and analytical $[---]$
2. Conclusion

Computer program-based developed on these mathematical relations allowed deformation draw diagrams for different types of contact pairs of materials, forces of pressure, size and roughness of contact surfaces. The analysis shows that deformation diagrams must necessarily remain in resilient and have the smallest value for the measurement to be accurate and not be affected Part measured. For this purpose the choice of force must be the result of measuring the balance between a force large enough to provide sufficient contact closely track the stylus and to avoid distortion in the surface roughness is only a low enough power to ensure that does not significantly affect strain measurement results. From the study results as it charts the force has a value between 0.05 and 0.3 daN.

3. Bibliography

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