Linear Motion Performed by a Double Hexapodal Robot

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ABSTRACT

The double hexapodal robot consists in two staged hexapodal platforms – Stewart Gough platforms - combining in a certain measure the advantages of the robots with parallel kinematics and of the serial robots: high accuracy, high stiffness, fast response and small dimensions, having an extended operating space. Different modeling and construction aspects were presented in few previous issues. The use of this system depends of the control software performance. An important class of motion is presented in this article: linear motion.

INTRODUCTION

The double hexapodal robot ROBEX is based on two Stewart-Gough platforms, having specific constructive parameters, an assembling procedure making it modular and having a particular condition imposed in order to reduce the redundancy of the system. The result is a more flexible positioning system. The direct and inverse kinematics of the platform together with new calibration methods were developed by numerous authors, among them being Raghavan [1] and Dietmaier [2]. The idea of the project ROBEX was to extend the operating space supplied by a single platform. The double structure assumes that both modules have identical configurations, hence the same control parameters.

THEORETICAL CONSIDERATIONS

To define the platform position three parameters are used: coordinates x, y, z of the platform centre; to define the orientation of the platform, three independent angles (Euler's angles) were introduced:

- ψ - the rotation angle around the fixed vertical axis
- θ - the tilt angle of the platform around a horizontal axis that rotates itself with angle ψ
- φ - the proper rotation angle of the platform around the axis that pass through its centre and is perpendicular on it.

In order to establish the vertexes coordinates of the hexagons B_{1i} and B_{2i}, $i = 1,..,6$, a matrices method is used, with the next notations:

$$\underline{\underline{V}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \underline{\underline{\psi}} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \underline{\underline{\theta}} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad \underline{\underline{\varphi}} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[V] being the positioning vector, and [ψ], [θ], [φ] the elementary rotation matrices [3].

The next coordinates systems are defined (figure 1): the fixed system $O_0x_0y_0z_0$, the system $O_1x_1y_1z_1$, linked to the intermediate mobile platform 1 and the system $O_2x_2y_2z_2$, linked to the final mobile platform 2.

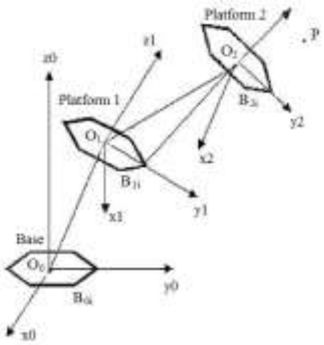

Figure 1: The double staged platform coordinates systems

Starting from the position and the orientation of the platform 2, described by its centre coordinates $x^0_{O_2}, y^0_{O_2}, z^0_{O_2}$ in the system $O_0x_0y_0z_0$ and the orientation angles $\psi^0_2 = \Psi, \theta^0_2 = \Theta, \varphi^0_2 = \Phi$ in the same system, the analogue parameters have to be determined for:

- the intermediate platform 1 in the system $O_0x_0y_0z_0$: $x^0_{O_1}, y^0_{O_1}, z^0_{O_1}$, respectively $\psi^0_1, \theta^0_1, \varphi^0_1$
- the final platform 2 in the system $O_1x_1y_1z_1$: $x^1_{O_2}, y^1_{O_2}, z^1_{O2}$, respectively $\psi^1_2, \theta^1_2, \varphi^1_2$

For a given point P we have the next relation between its coordinates in vector representation V^0_P, V^1_P, V^2_P in the systems $O_0x_0y_0z_0$, $O_1x_1y_1z_1$, respectively $O_2x_2y_2z_2$:

$$V^0_P = V^0_{O_1} + \psi^0_1 \cdot \theta^0_1 \cdot \varphi^0_1 \cdot V^1_P \quad (1)$$

$$V^1_P = V^1_{O_2} + \psi^1_2 \cdot \theta^1_2 \cdot \varphi^1_2 \cdot V^2_P \quad (2)$$

$$V^0_P = V^0_{O_2} + \psi^0_2 \cdot \theta^0_2 \cdot \varphi^0_2 \cdot V^2_P = V^0_{O_2} + \Psi \cdot \Theta \cdot \Phi \cdot V^2_P \quad (3)$$

For identical configurations [4], we have identical distances $O_0O_1 = O_1O_2$, namely:

$$V^0_{O_1} = V^1_{O_2} \quad (4)$$

and identical angles

$$\psi^0_1 = \psi^1_2 = \psi, \theta^0_1 = \theta^1_2 = \theta, \varphi^0_1 = \varphi^1_2 = \varphi \quad (5)$$

Replacing the relation (2) in (1) and taking into account of (4) and (5), we obtain:

$$\begin{aligned} V^0_P &= V^0_{O_1} + \psi \cdot \theta \cdot \varphi \cdot \left(V^1_{O_2} + \psi \cdot \theta \cdot \varphi \cdot V^2_P\right) = \\ &V^0_{O_1} + \psi \cdot \theta \cdot \varphi \cdot \left(V^0_{O_1} + \psi \cdot \theta \cdot \varphi \cdot V^2_P\right) = \\ &V^0_{O_1} + \psi \cdot \theta \cdot \varphi \cdot V^0_{O_1} + \left(\psi \cdot \theta \cdot \varphi\right)^2 \cdot V^2_P \end{aligned} \quad (6)$$

Considering the relation (3), we obtain the identity:

$$V^0_{O_1} + \psi \cdot \theta \cdot \varphi \cdot V^0_{O_1} + \left(\psi \cdot \theta \cdot \varphi\right)^2 \cdot V^2_P = V^0_{O_2} + \Psi \cdot \Theta \cdot \Phi \cdot V^2_P \quad (7)$$

On the other hand

$$V^0_{O_2} = V^0_{O_1} + \psi \cdot \theta \cdot \varphi \cdot V^1_{O_2} = V^0_{O_1} + \psi \cdot \theta \cdot \varphi \cdot V^0_{O_1} \quad (8)$$

Replacing (8) in (7), we have

$$(\psi \cdot \theta \cdot \varphi)^2 \cdot V_P^2 = \Psi \cdot \Theta \cdot \Phi \cdot V_P^2 \qquad (9)$$

or

$$(\psi \cdot \theta \cdot \varphi)^2 = \Psi \cdot \Theta \cdot \Phi \qquad (10)$$

The final relation results:

$$\psi \cdot \theta \cdot \varphi = \sqrt{\Psi \cdot \Theta \cdot \Phi} \qquad (11)$$

Using the same method, the relation (11) can be generalized for a system with *n* modules:

$$\psi \cdot \theta \cdot \varphi = \sqrt[n]{\Psi \cdot \Theta \cdot \Phi} \qquad (12)$$

These results are presented in [4] and [5].
The coordinates of the points B_{1i} and B_{2i}, $i = 1,..,6$ are then determined:

$$V_{B_{2i}}^0 = V_{O_2}^0 + \Psi \cdot \Theta \cdot \Phi \cdot V_{B_{2i}}^2 \qquad (13)$$

$$V_{B_{1i}}^0 = V_{O_1}^0 + \psi \cdot \theta \cdot \varphi \cdot V_{B_{1i}}^1 \qquad (14)$$

From the relations (13) and (14) the lengths of the legs are deduced, namely the control parameters.

$$V_{O_2}^0 = V_S^0 - V_{O_2}^0 + \Psi \cdot \Theta \cdot \Phi \cdot V_S^2 \qquad (15)$$

MODELLING AND CONTROL

The control and simulation software of the ROBEX was performed in LabVIEW. In figure 2, different configurations of the double hexapodal structure are shown (in x-y projection in the left graph and in x-z projection in the right graph); the angles Ψ, Θ, Φ are the angles defined in the previous paragraph, the angles that determine the orientation of the final platform:

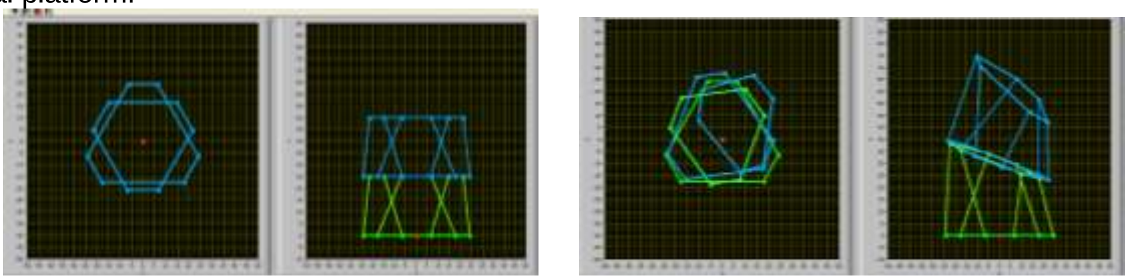

Figure 2: Position and orientation of the double structure

The controls placed on the Control Panel of this application allow modifying the shape of the irregular hexagon by the aid of two parameters: the radius of the circumscribed circle and the angle determined by the centre and two neighbouring vertexes. The flexibility of the program can be easily noticed comparing the difference between the sizes and geometry of the irregular hexagons from figure 2 and from figure 3:

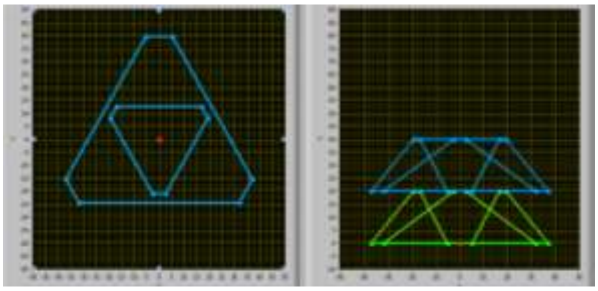

Figure 3: Larger sizes of the system and different angles

CONSTRUCTION

The construction of ROBEX is based on the LM 1247 linear electrical actuator produced by FAULHABER (figure 4), having 6 μm theoretical resolution. The length of a leg is about 200 mm, with a stroke of 20 mm.

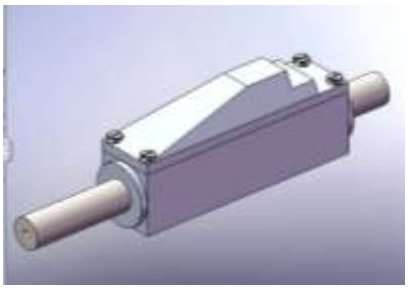

Figure 4: Linear motor LM 1241

From practical considerations, the next solution was preferred: the lower and upper joints are universal joints; between the motor shaft and the upper universal joint a radial bearing is placed. In this case, the six legs represents 18 elements and the mobile platform the 19^{th}, namely n = 19. So, there are:

- 12 universal joint of class 4
- 6 translation joints of class 5
- 6 rotation joints of class 5

The degree of mobility for one hexapod is:

$$M = 6 \cdot 19 - (4 \cdot 12 + 5 \cdot 6 + 5 \cdot 6) = 114 - (48 + 30 + 30) = 114 - 108 = 6 \qquad (16)$$

In order to assure a greater mobility of the double system, the "classical" shape of the platform (figure 5) was modified, obtaining the "adapted shape", with an important difference between the diameters of the upper and the lower platforms (figure 6).


Fig. 5: Classical platform


Fig. 6: Adapted platform

Special designed connecting elements are used for modular assembling (figure 7); another function of these elements is to assure the co planarity between the centres of the universal joints from the base platform of the upper module and of the universal joints from the mobile platform of the lower module.


Figure 7: Double hexapodal robot ROBEX

The functional model, in a simplified representation, contains:

- the mechanical structure
- the actuating section, including the power supply
- the external controller (a PC or equivalent) and the control software – a LabVIEW application.

The controller generates the positioning algorithms and the motion paths, which are transmitted to the actuating section, based on 12 LM1247-020-01 motors and MCLM 3003/06S motion controllers that carries out the displacement of the mechanical part. The PC is linked to the actuating section by a RS232 serial interface, using a set of controls in ASCII format.

LINEAR MOTION PERFORMED BY ROBEX

The utility of this system depends of the complexity of the motion which can be generated by the control module. Linear path is one of the most used motion, for instance in the drilling operation. The equation of a line of direction defined by the director parameters *l, m, n* and passing through a point of coordinates x_0, y_0, z_0 is:

$$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n} \qquad (17)$$

Unfortunately, neither the parameters *l, m, n,* nor the angles ψ,θ,φ defined in a previous paragraph are not sufficient of intuitive to express a direction. Therefore, another approach is preferred: a hole is defined by the two extreme points, the contact point C (x_C, y_C, z_C) between tool and the surface to be drilled and the terminal point of the hole T (x_T, y_T, z_T). However, the parameters mentioned above are necessary to be introduced in the control program.

Assuming that the positioning system is in the default position, with the characteristic point V (the vertex of the pyramid in fig. 8) in point H of coordinates *(0,0,h),* we have to move the system to point C, preparing in the same time the required orientation, namely the final platform must be perpendicular on the hole axis. Then, after the tool – part contact is attained, the system has to perform a linear motion between points C and T, maintaining the orientation unchanged.

In this way, there are two sequences of the motion of the point V:

1. along the segment HC with variable orientation (figure 9)
2. along the segment CT with constant orientation (figure 10) – technological sequence.

In reality the program contains the reverse motion too.

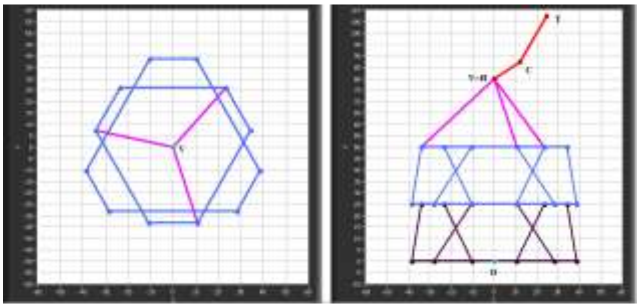

Figure 8: Home position

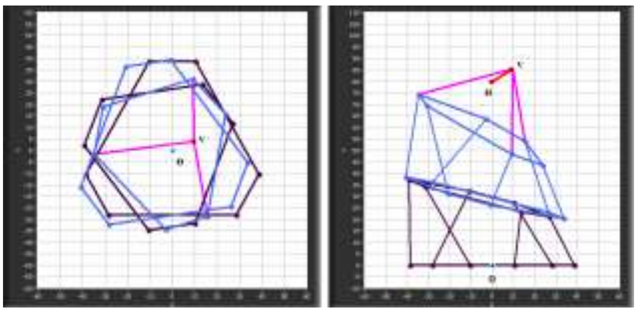

Figure 9: Sequence HC of the tool positioning

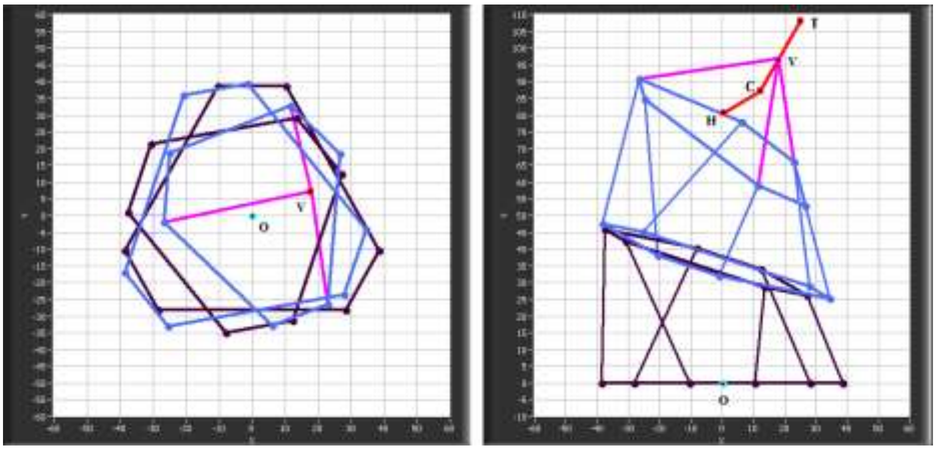

Figure 10: Technological sequence CT of drilling

In the figures 8, 9 and 10, the path is represented with red line.

The modeling and control program is based on the next relations:

1. To establish the control angles ψ,θ,φ, first we write the equation of the straight line passing through the points C and T, where k is a real parameter:

$$\frac{x-x_C}{x_T-x_C}=\frac{y-y_C}{y_T-y_C}=\frac{z-z_C}{z_T-z_C}=k \quad (18)$$

From (17) and (18) we can determine the director parameters *l, m, n* of the segment CT:

$$\begin{cases} l=\dfrac{x_T-x_C}{\sqrt{(x_T-x_C)^2+(y_T-y_C)^2+(z_T-z_C)^2}} \\ m=\dfrac{y_T-y_C}{\sqrt{(x_T-x_C)^2+(y_T-y_C)^2+(z_T-z_C)^2}} \\ n=\dfrac{z_T-z_C}{\sqrt{(x_T-x_C)^2+(y_T-y_C)^2+(z_T-z_C)^2}} \end{cases} \quad (19)$$

Knowing that

$$\begin{cases} l=\sin\theta\cos\psi \\ m=\sin\theta\sin\psi \\ n=\cos\theta \end{cases} \quad (20)$$

We obtain

$$\begin{cases} \psi=\arctan\dfrac{m}{l}=\arctan\dfrac{y_T-y_C}{x_T-x_C} \\ \theta=\arccos n=\arccos\dfrac{z_T-z_C}{\sqrt{(x_T-x_C)^2+(y_T-y_C)^2+(z_T-z_C)^2}} \end{cases} \quad (21)$$

Note: Proper rotation angle φ doesn't matter in the case of a drilling operation.

2. To move the system after the line CT with the constant orientation determined above, it is useful to write the equation (18) of CT in parametric form:

$$\begin{cases} x = x_C + k \cdot (x_T - x_C) \\ y = y_C + k \cdot (y_T - y_C) \\ z = z_C + k \cdot (z_T - z_C) \end{cases} \qquad (22)$$

To obtain the control coordinates *x, y, z* in a multitude of points, necessary for modeling and control, them calculus algorithm was introduced in a "For" loop, the parameter k from (22) being a linear function of *I,* where *I* is the index of the loop. Thus, figures 8, 9 and 10 represents only frames from an animation modeling software.

Important note: In the general case of a certain curve, the parametric equations (22), corresponding to a straight line, can be replaced with the parametric equation of that curve. The path of the curve is generated in a similar mode.

CONCLUSIONS

In this article the control and modeling software used to generate a linear path with the double hexapodal robot ROBEX is presented. Using the LabVIEW features, graphical representation of the systems motion is shown in an intuitive mode. The linear motion is governed by a set of equations, also deduced and presented in a concise form.

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