SIMPLE KALEIDOSCOPE DESIGN FOR REFLECTANCE MEASUREMENT
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Abstract
In this paper we describe simple design of a kaleidoscope used for measuring the bidirectional reflectance distribution function with aim to obtain the bidirectional texture function of various surface samples. The first prototype of the kaleidoscope was designed in symmetrical configuration with four tapered mirror walls. We discuss the influence of different design parameters with respect to expected results. The proposed kaleidoscope was manufactured and its performance evaluated.

Keywords
kaleidoscope, mirror design, reflectance measurement

Introduction
Fast and accurate measurement of bidirectional reflectance distribution function (BRDF)¹ used to model various material appearance finds its application in many technical branches involving computer vision and computer graphics. The acquisition, processing and further modelling of real world appearance requires special gantries and is both time and cost demanding as BRDF is a four dimensional function for monospectral data. Since the BRDF is defined for a single point, there is its extension representing a spatial variation known as the Bidirectional Texture Function (BTF ). Such representation can be obtained by mathematical processing of a number of sample images (in fact surface reflectances) taken under many different combinations of incident illumination and observation angles in macroscopic and mezoscopic scale. As mentioned, acquisition of this data set can be quite time consuming as about ten(s) of thousands of surface images need to be taken and processed. The whole scanning process with sufficient spatial and directional discretization is traditionally measured using stationary measurement gantries²³⁴. These instruments unfortunately do not enable sampling of larger specimens or those which may change its pattern when they are moved. Making the measurement instrument portable⁵⁶⁷ may resolve these issues. The aim of our work is to design a portable kaleidoscope that is suitable for BTF measurement. A multidirectional view of the sample at once thanks to the principle of a kaleidoscope can significantly reduce the acquisition time and the utility of the measurement instrument.

Design strategy
For the purpose of this paper, we will assume we know what our “holy grail” is, i.e. the “best BTF” acquisition. Then, we can define a quality function that describes how close the BTF captured by our kaleidoscope is to the “best BTF”. Naturally, we want to cover maximum number of combinations of the illumination angles φᵢ, θᵢ and observation angles φₒ, θₒ. Illumination angles are changed by controlling the direction of the beam leaving a projector (see fig. 1).

In the first place, we must carefully inspect variable and dependent parameters of the kaleidoscope. The kaleidoscope will not work alone and independently: a camera lens and a full frame CCD chip will form an optical system for an image capture. The optical output of the kaleidoscope must match the aperture (f-number, f/#) and field of view (FOV) of this system.
Figure 1: Kaleidoscope layout: scheme, dimensions, unfolding

As the kaleidoscope is a set of plane mirrors, it creates a set of images. These images are formed on a sphere, which can be found by unfolding technique (shown in fig. 1). The radius $R$ of this sphere is given by the size of the kaleidoscope base and its taper angle $\alpha$. The camera FOV limits the number of images captured, i.e. the number of reflections of an image forming ray, and consequently the maximal observation angle $\phi_o$.

As the images being captured are not in a plane, we must consider the depth of field. A depth of field (DOF) depends on a camera focal length $f'$, a distance $x$ of the object being in perfect focus and a camera f-number. Objects at distances $x_o$ (farthest) and $x_b$ (closest) are still in focus as their circles of confusion are smaller or equal to a pixel size (variables explained in fig. 2).
Figure 2: Depth of field of a lens; D – clear aperture, ξ – object plane, ξ’ – image (detector) plane

DOF is calculated using Newton transfer equations and equations for similar triangles:

\[ x \cdot x' = -f^2 \]
\[ x_b \cdot x'_b = -f'^2 \]
\[ x_d \cdot x'_d = -f'^2 \]  

(1)

\[ \frac{D}{f' + x'_b} = \frac{\text{pixel size}}{x'_b - x'} \]
\[ \frac{D}{f' + x'_d} = \frac{\text{pixel size}}{x' - x'_d} \]  

(2)

\[ f / \# = \frac{f'}{D} \]  

(3)

\[ \text{DOF} = x_b - x_d \]  

(4)

x is the distance at what the lens is focused, x’ is the position of the detector. Ways to increase the DOF together with their advantages and drawbacks are shown in table 1.

<table>
<thead>
<tr>
<th>mechanism</th>
<th>pros</th>
<th>cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>increase of x</td>
<td>simple</td>
<td>leads to bulky mechanical constructions, unwanted for portable devices</td>
</tr>
<tr>
<td>increase of f/#</td>
<td>very easy</td>
<td>limited by diffraction</td>
</tr>
<tr>
<td>decrease of f'</td>
<td>can be chosen from the off-the-shelf camera lenses</td>
<td>restricted by the choice of available lenses, higher aberrations for short f' (or very expensive lenses)</td>
</tr>
</tbody>
</table>

Another way to decrease the demand to a high DOF is to use a lower taper angle. In the special case when \( \alpha = 0 \), images formed by the kaleidoscope are within a plane, therefore DOF = 0mm. Also all images are in full original size, not shrunk as it happens for another taper angles. Unfortunately, the maximal observation angle \( \phi_o \) then equals to the field angle (FOV/2) of used camera. If a sub-picture is shrunk, its spatial resolution decreases. On the other hand, the shrinkage of images for high observation angles are only natural and accord with a real situation.
**Parameter analysis**

Mechanical parameters of the kaleidoscope are: length, taper angle, number of sides, base size, camera focal length, f-number and FOV, distance between the camera and the top of the kaleidoscope, chip size, pixel size. Other parameters that must be somehow chosen are: shape and size of a sample, number of reflections at kaleidoscope mirrors in meridional plane.

The following table briefly summarizes the influence of each parameter. However, one must take into account all of them together. For example, a number of reflections together with a taper angle and base size determine the necessary DOF. When increasing DOF by closing the camera aperture, we must consider not only diffraction but also an exposure time: as thousands of pictures are to be taken, we cannot afford long exposure times.

<table>
<thead>
<tr>
<th>parameter</th>
<th>why to decrease</th>
<th>why to increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>kaleidoscope length</td>
<td>mechanical stability and portability</td>
<td>higher number of reflections, easier to keep images in focus</td>
</tr>
<tr>
<td>number of reflections</td>
<td>to keep DOF small</td>
<td>to capture more images in once</td>
</tr>
<tr>
<td>taper angle</td>
<td>achievable angles $\phi_0$ are interrupted by gaps of $2\alpha$; to keep DOF small</td>
<td>easier and faster way to reach high $\phi_0$</td>
</tr>
<tr>
<td>number of sides</td>
<td>mechanically easier, less edges that cause blind lines</td>
<td>observation angles $\theta_0$ are more evenly distributed</td>
</tr>
<tr>
<td>base size</td>
<td>more sub-pictures in one picture</td>
<td>decreases DOF, more data in each sub-picture</td>
</tr>
<tr>
<td>camera f'</td>
<td>short capture distance, large FOV</td>
<td>lower demands on corrections of aberrations</td>
</tr>
<tr>
<td>f/#</td>
<td>more light on the detector, negligible diffraction</td>
<td>longer DOF, decreases optical aberrations</td>
</tr>
<tr>
<td>top of kaleidoscope –</td>
<td>to increase the field angle, to utilize the camera FOV and the chip area</td>
<td>to make enough room for a beam splitter</td>
</tr>
<tr>
<td>camera distance</td>
<td>price</td>
<td>higher resolution</td>
</tr>
<tr>
<td>chip size</td>
<td>increases resolution for a fixed f’</td>
<td>better sensitivity and therefore shorter exposure times</td>
</tr>
</tbody>
</table>

**Simulation software**

We have developed a software system for kaleidoscope device optimization. It computes a picture that would be taken by a system with chosen parameters. All above mentioned parameters can be controlled by user. The programme also gives the observation angles for each sub-picture (a direct view, after one reflection, two reflections etc.). For maximal use of the chip area, it is wise to choose the number of kaleidoscope sides and the base shape so as the chip area is covered without blank parts. Equilateral triangle for three sides, square for four and hexagon for six sides do this job well. An example of the simulation output is shown in fig. 3. The graphs in the second row show observation angles for all sub-images. Ideally, the crosses should fill up the circles evenly which would mean that we covered the observation hemisphere well.

For a triangle and a hexagon, many of the sub-pictures are fragmented, i.e. we can see only a part of the sample. The rest of the triangle or hexagon in the appearing sub-pictures belongs to another observation angle (was formed by other reflections). Only with a square, all sub-pictures are full images of the sample (no fragmentation).
Figure 3: SW output for 3, 4 and 6 side kaleidoscope. Blue parts were formed by more than 6 reflections. Red lines are mirror edges. Sample shape is hexagonal. All parameters but the number of sides are the same.

A simple prototype

To be able to verify that our SW gives valid output, we made a very simple prototype of a kaleidoscope. The prototype has four sides, the mirrors are rectangles. We made the taper angle adjustable in a small span: The bottom edges are held to the base by a tape, which forms a kind of hinge. At the top, there is a diaphragm that encircles the mirrors. By closing this diaphragm, we bring the tops of the mirrors closer together – the taper angle decreases.

Figure 4: The kaleidoscope prototype, left; a picture taken with the prototype, right.
Pictures of the prototype and the image taken with the kaleidoscope are shown in fig. 4. It is obvious that the quality of the mirror edges influences the picture. In our case, the edges were covered by a black tape which causes quite wide black stripes in the picture. Anyway, the prototype showed to be suitable for proving that our software simulator works correctly.

Conclusion

We analysed parameters of a kaleidoscope and their relationships. We are developing a quality function that will help us to choose an optimal kaleidoscope design. To see the optical behaviour of kaleidoscopes with different parameters, we developed a program where these parameters can be controlled. A simple prototype was engineered to check the outputs of the programme and to reveal possible mechanical problems of this type of BTF measuring device. The next step will be to finalize the quality function and determine the parameters of the device.

Acknowledgements

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References